# Combined Effect of QCD Resummation and QED Radiative Correction to $W$ boson Mass Measurement at the LHC[*]

*Qing-Hong Cao and C.-P. Yuan*

Department of Physics and Astronomy, Michigan State University, USA

### Abstract

A precise determination of the $W$ boson mass at the CERN LHC requires a theoretical calculation in which the effect of the initial-state multiple soft-gluon emission and the final-state photonic correction are simultaneously included . Here, we present such a calculation and discuss its prediction on a few most relevant distributions of the decay leptons.

## 1. INTRODUCTION

As a fundamental parameter of the Standard Model (SM), the mass of the $W$-boson ($M_W$) is of particular importance. Aside from being an important test of the SM itself, a precision measurement of $M_W$, together with an improved measurement of top quark mass ($M_t$), provides severe indirect bounds on the mass of Higgs boson ($M_H$). With a precision of 15 MeV for $M_W$ [1] and 2 GeV for $M_t$ at the LHC [2], $M_H$ in the SM can be predicted with an uncertainty of about 30% [1]. Comparison of these indirect constraints on $M_H$ with the results from direct Higgs boson searches, at the LEP2, the Tevatron and the CERN Large Hadron Collider (LHC), will be an important test of the SM. In order to have a precision measurement of $M_W$, the theoretical uncertainties, dominantly coming from the transverse momentum of the $W$-boson ($P_T^W$), the uncertainty in parton distribution function (PDF) and the electroweak (EW) radiative corrections to the $W$ boson decay, must be controlled to a better accuracy [1, 3].

At the LHC, most of $W$ boson is produced in the small transverse momentum region. When $P_T^W$ is much smaller than $M_W$, every soft-gluon emission will induce a large logarithmic contribution to the $P_T^W$ distribution so that the order-by-order perturbative calculation in the theory of Quantum chromodynamics (QCD) cannot accurately describe the $P_T^W$ spectrum and the contribution from multiple soft-gluon emission, which contributes to all orders in the expansion of the strong coupling constant $\alpha_s$, needs to be summed to all orders. It has been shown that by applying the renormalization group analysis, the multiple soft-gluon radiation effects can be resummed to all orders to predict the $P_T^W$ distribution that agrees with experimental data [4, 5]. RESBOS, a Monte Carlo (MC) program [5] resumming the initial-state soft-gluon radiations of the hadronically produced lepton pairs through EW vector boson production and decay at hadron colliders $p\bar{p}/pp \rightarrow V(\rightarrow \ell_1\bar{\ell}_2)X$, has been used by the CDF and DØ Collaborations at the Tevatron to compare with their data in order to determine $M_W$. However, RESBOS does not include any higher order EW corrections to describe the vector boson decay. The EW radiative correction, in particular the final-state QED correction, is crucial for precision measurement of $W$ boson mass at the Tevatron, because photon emission from the final-state charged lepton can significantly modify the lepton momentum which is used in the determination of $M_W$. In the CDF Run Ib $W$ mass measurement, the mass shifts due to radiative effects were estimated to be $-65 \pm 20$ MeV and $-168 \pm 10$ MeV for the electron and muon channels, respectively [6]. The full next-to-leading order (NLO) $O(\alpha)$ EW corrections have been calculated [7, 8] and resulted in WGRAD [8], a MC program for calculating $O(\alpha)$ EW radiative corrections to the process $p\bar{p} \rightarrow \nu_\ell\ell(\gamma)$. However, WGRAD does not include the dominant correction originated from the initial-state multiple soft-gluon emission. To incorporate both the initial-state QCD and and final-state QED corrections into a parton level MC
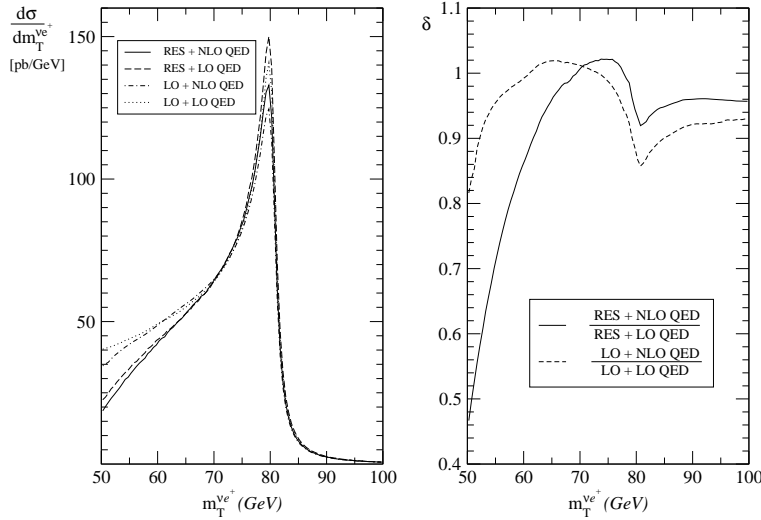
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Fig. 1: Transverse mass distribution of $W^+$ boson

program is urgently required to reduce the theoretical uncertainties in interpreting the experimental data at the Tevatron. It was shown in Refs. [7, 8] that at the NLO, the EW radiative correction in $p\bar{p} \to \ell\nu_l(\gamma)$ is dominated by the final-state QED (FQED) correction. Hence, in this paper we present a consistent calculation which includes both the initial-state multiple soft-gluon QCD resummation and the final-state NLO QED corrections, and develop an upgraded version of the RESBOS program, called RESBOS-A [†], to simulate the signal events. Here, we only present the phenomenological impacts on a few experimental observalbes, the transverse mass of $W$ boson ($M_T^W$) and the transverse momentum of charged lepton ($p_T^\ell$), that are most sensitive to the measurement of $M_W$. We focus our attention on the electron lepton only, though our analysis procedure also applies to the $\mu$ lepton. The detailed formula, the SM inputting parameters and the kinematics cuts are given in Ref. [9].

## 2. PRECISION MEASUREMENT OF W MASS

In Fig. 1, we show various theory predictions on the $M_T^W$ distribution. The legend of the figure is defined as follows:

- LO : including only the Born level initial-state contribution,
- RES : including the initial-state multiple soft-gluon corrections via QCD resummation,
- LO QED : including only the Born level final-state contribution,
- NLO QED : including the final-state NLO QED corrections.

For example, the solid curve (labelled as RES+NLO QED) in Fig. 1(a) is the prediction from our combined calculation, given by Eqs. (1) and (2) of Ref [9].

As shown in Fig. 1(a), compared to the lowest order cross section (dotted curve), the initial state QCD resummation effects (dashed curve) increase the cross section at the peak of the $M_T^W$ distribution by about 6%, and the final state NLO QED corrections (dot-dashed curve) decrease it by about $-12\%$, while the combined contributions (solid curve) of the QCD resummation and FQED corrections reduce it by 6%. In addition to the change in magnitude, the line-shape of the $M_T^W$ distribution is significantly modified by the effects of QCD resummation and FQED corrections. To illustrate this point, we plot the ratio of the (RES+NLO QED) differential cross sections to the LO ones as the solid curve in Fig. 1(b). The dashed curve is for the ratio of (LO+NLO QED) to LO. As shown in the figure, the QCD resummation effect dominates the shape of $M_T^W$ distribution for $65\,\mathrm{GeV} \le \mathrm{M_W} \le 95\,\mathrm{GeV}$, while the FQED

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[†]A Fortran code that implemets the theoretical calculation presented in this work.

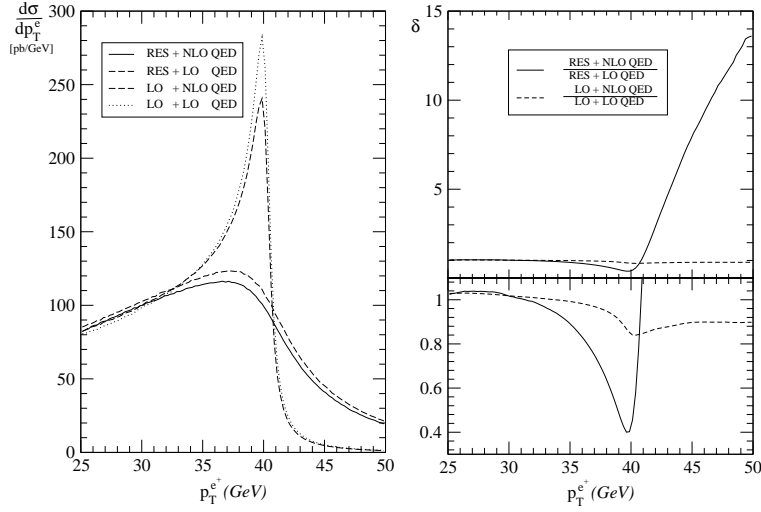Fig. 2: Transverse momentum distributions of $e^+$

correction reaches its maximal effect around the Jacobian peak ($M_T^W \simeq M_W$). Hence, both corrections must be included to accurately predict the distribution of $M_T^W$ around the Jacobian region to determine $M_W$. We note that after including the effect due to the finite resolution of the detector (for identifying an isolated electron or muon), the size of the FQED correction is largely reduced [7, 8].

Although the $M_T^W$ distribution is an optimal observable for determining $M_W$ at the LHC with a low luminosity, it requires an accurate measurement of the missing transverse momentum direction which becomes more difficult to control with a high luminosity option (when multiple scattering becomes important). On the other hand, the transverse momentum of the decay charged lepton ($p_T^e$) is less sensitive to the detector resolution, so that it can be used to measure $M_W$ and provide important cross-check on the result derived from the $M_T^W$ distribution, for they have different systematic uncertainties. Another important feature of this observable is that $p_T^e$ distribution is more sensitive to the transverse momentum of $W$ boson. Hence, the QCD soft-gluon resummation effects, the major source of $p_T^W$, must be included to reduce the theoretical uncertainty of this method. In Fig. 2(a), we show the $p_T^e$ distributions predicted by various theory calculations, and in Fig. 2(b), the ratios of the higher order to lowest order cross sections as a function of $p_T^e$. The lowest order distribution (dotted curve) shows a clear and sharp Jacobian peak at $p_T^e \simeq M_W/2$, and the distribution with the NLO final-state QED correction (dot-dashed curve) also exhibits the similar Jacobian peak with the peak magnitude reduced by about $15\%$. But the clear and sharp Jacobian peak of the lowest order and NLO FQED distributions (in which $p_T^W = 0$) are strongly smeared by the finite transverse momentum of the $W$ boson induced by multiple soft-gluon radiation, as clearly demonstrated by the QCD resummation distribution (dashed curve) and the combined contributions of the QCD resummation and FQED corrections (solid curve). Similar to the $M_T^W$ distribution, the QCD resummation effect dominates the whole $p_T^e$ range, while the FQED correction reaches it maximum around the Jacobian peak (half of $M_W$). The combined contribution of the QCD resummation and FQED corrections reaches the order of $45\%$ near the Jacobian peak. Hence, these lead us to conclude that the QCD resummation effects are crucial in the measurement of $M_W$ from fitting the Jacobian kinematical edge of the $p_T^e$ distribution.

In order to study the impact of the presented calculation to the determination of the $W$ boson mass, the effect due to the finite resolution of the detector should be included, which will be presented elsewhere.

## 3. ACKNOWLEDGEMENTS

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